LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **JULY 2025**

UMT 5501 – REAL ANALYSIS - II

Date: 11-07-2025 Dept. No. Max. : 100 Marks

Time: 10:00 AM - 01:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Answer the following**
a)	Define limit of a function.
b)	Give an example of a continuous function.
c)	Define differentiability of a function.
d)	State a property of Riemann integration.
e)	Define compact set in $\mathbb{R}$.
2.	**Fill in the blanks**
a)	The cluster point of the set $\{1, \frac{1}{2}, \frac{1}{3}, \cdots\}$ is ………………………
b)	$\lim_{x \to 2} \left(\frac{x^3 - 7x}{4x^2 - 5x}\right) = \ldots\ldots$
c)	If f is continuous on a closed interval $I = [a, b]$, and that f has a derivative in (a, b), then there exists at least one point $c \in (a, b)$ such that $f'(c) =$ ………………
d)	The norm of the partition $(0, 1, 2, 4)$ of the interval $[0, 4]$ is ……….
e)	An arbitrary intersection of closed sets in $\mathbb{R}$ is ……… in $\mathbb{R}$.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Choose the correct answer for the following**
a)	$\lim_{x \to 0} \frac{\sin x}{x}$ is i) 0 ii) 1 iii) π iv) ∞
b)	Let $A \subseteq \mathbb{R}$ and let $f: A \to \mathbb{R}$. If there exists a constant $K > 0$ such that $\lvert f(x) - f(u)\rvert \leq K \lvert x - u\rvert$, for all $x, u \in A$, then f is said to be a ……… function on A. i) Lipschitz ii) uniformly continuous iii) continuous iv) differentiable
c)	The function $f: [-2, 2] \to \mathbb{R}$ defined by $f(x) = \lvert x\rvert$ is i) continuous at 0 ii) not differentiable at 0 iii) uniformly continuous on $[-2, 2]$ iv) all the above
d)	A constant function defined on an interval $[a, b]$ is i) Riemann integrable ii) Continuous iii) differentiable iv) all the above
e)	The set $G = \{x \in \mathbb{R}: 0 < x < 1\}$ is i) an empty set ii) an open set iii) a closed set iv) neither an open nor a closed set
4.	**State True or False**
a)	$\lim_{x \to 0} \frac{1}{x}$ exists in $\mathbb{R}$.
b)	The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is continuous at every point of $\mathbb{R}$.
c)	A function differentiable at a point is continuous at the point.
d)	If $f \in \mathcal{R}[a, b]$ and if $[c, d] \subseteq [a, b]$, then the restriction of f to $[c, d]$ is in $\mathcal{R}[c, d]$.
e)	Every closed subset of $\mathbb{R}$ is open in $\mathbb{R}$.

SECTION B - K3 (CO2)		
	Answer any TWO of the following	**(2 x 10 = 20)**
5.	State and prove the sequential criterion for limits.	
6.	State and prove uniform continuity theorem.	
7.	Prove that if $f \in \mathcal{R}[a,b]$ then f is bounded on $[a,b]$.	
8.	State and prove the preservation of compactness theorem.	
SECTION C – K4 (CO3)		
	Answer any TWO of the following	**(2 x 10 = 20)**
9.	If $f: A \to \mathbb{R}$ and if c is a cluster point of A, prove that f can have only one limit at c.	
10.	Let I be a closed bounded interval and let $f: I \to \mathbb{R}$ be continuous on I. Prove that f has an absolute maximum and an absolute minimum on I.	
11.	State and prove the Caratheodory's theorem.	
12.	Defend the following statements: i) The union of an arbitrary collection of open subsets in $\mathbb{R}$ is open in $\mathbb{R}$. ii) The union of an arbitrary collection of closed sets in $\mathbb{R}$ need not be closed in $\mathbb{R}$.	
SECTION D – K5 (CO4)		
	Answer any ONE of the following	**(1 x 20 = 20)**
13.	Let $I \subseteq \mathbb{R}$, let f and g be functions on I to $\mathbb{R}$. If f and g are differentiable at c, then prove that $f + g,\ fg, bf$ and $\frac{f}{g}$ (if $g(x) \neq 0,\ \forall\, x \in I$) are also differentiable at c. Further, prove that i. $(f + g)'(c) = f'(c) + g'(c)$ ii. $(f g)'(c) = f'(c)g(c) + g'(c)f(c)$ iii. $(bf)'(c) = bf'(c)$ iv. $\left(\frac{f}{g}\right)'(c) = \frac{g(c)f'(c)-f(c)g\prime(c)}{[g(c)]^2}$ if $g(c) \neq 0$	
14.	(a) State and prove Location of roots theorem. (b) State and prove Squeeze theorem on Riemann integration.	(12 marks) (8 marks)
SECTION E – K6 (CO5)		
	Answer any ONE of the following	**(1 x 20 = 20)**
15.	(a) Justify in detail using the concept of limit of a function that $\lim_{x \to c} x^2 = c^2$. (b) If $f: [a,b] \to \mathbb{R}$ is continuous on $[a,b]$, establish that $f \in \mathcal{R}[a,b]$.	(10 marks) (10 marks)
16.	(a) State and prove Taylor's theorem. (b) Prove that a subset of $\mathbb{R}$ is compact if and only if it is bounded and closed.	(12 marks) (8 marks)

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